EX NAVODAYAN FOUNDATION
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Full Syllabus

JEE-Main

Paper-5

Test Date: **M.M:300**

TEST INSTRUCTIONS

1. The test is of **3 hours** duration.
2. The test booklet consists of **75 questions**.
3. The maximum marks are **300**.
4. All questions are compulsory.
5. There are three parts in the questions paper consisting of Physics, Chemistry and Mathematics having **25 questions in each part**.

Each Parts Contains –

- 20 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. All questions are carrying **+4 marks** for right answer and **-1 mark** for wrong answer.
- 05 questions with answer as **numerical value** all questions are carrying **+4 marks** for right answer and **–1 marks** for wrong answers.

Name of the Candidate (in Capital Letter): _____

Registration No. _____

Invigilator Signature

_____

Physics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. In an expression $a \times 10^b$:

   (a) a is order of magnitude for $b \le 5$ (b) b is order of magnitude for $a \le 5$

   (c) b is order of magnitude for $a \ge 5$ (d) b is order of magnitude for $5 < a \le 10$

2. Applying the principle of homogeneity of dimensions, determine which one is correct, where T is the time period, G is gravitational constant, m is mass, r is radius of orbit.

   (a) $T^2 = \frac{4\pi^2 r}{GM^2}$ (b) $T^2 = 4\pi^2 r^3$ (c) $T^2 = \frac{4\pi^2 r^3}{GM}$ (d) $T^2 = \frac{4\pi^2 r^2}{GM}$

3. Two vectors $\vec{x}$ and $\vec{y}$ have equal magnitude. The magnitude of $(\vec{x}-\vec{y})$ is n times the magnitude of $(\vec{x}+\vec{y})$. The angle between $\vec{x}$ and $\vec{y}$ is

   (a) $\cos^{-1}\left(\frac{-n^2-1}{n^2-1}\right)$ (b) $\cos^{-1}\left(\frac{n^2+1}{-n^2-1}\right)$ (c) $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$ (d) $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$

4. What will be the projection of vector $\vec{A} = \hat{i}+\hat{j}+\hat{k}$ on vector $\vec{B} = \hat{i}+\hat{j}$

   (a) $\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$ (b) $(\hat{i}+\hat{j})$ (c) $\sqrt{2}(\hat{i}+\hat{j})$ (d) $2(\hat{i}+\hat{j}+\hat{k})$

5. Two projectile P_1 and P_2 thrown with speed in the ratio $\sqrt{3}:\sqrt{2}$ attain the same height during their motion. If P_2 is thrown at an angle of 60° with the horizontal, the angle of projection of P_1 with horizontal will be

   (a) 37° (b) 53° (c) 45° (d) 60°

6. A body at rest in moving along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to:

   (a) $t^{3/2}$ (b) $t^{1/2}$ (c) $t^{1/4}$ (d) $t^{3/4}$

7. A 2 kg steel rod of length 0.6 m is clamped on a table vertically at its lower end and is free to rotate in vertical plane. The upper end is pushed so that the rod falls under gravity, ignoring the friction due to clamping at its lower end, the speed of the free end of rod when it passes through its lowest position is (g = 10 m/s)

   (a) 3 m/s (b) 6 m/s (c) 4 m/s (d) 6.3 m/s

8. A small ball of mass 'm' and density 'ρ' is dropped in a viscous liquid of density ρ_0. After sometime, the ball falls with a constant velocity. The viscous force on ball is:

   (a) $mg\left(1-\frac{\rho_0}{\rho}\right)$ (b) $mg(1-\rho\rho_0)$ (c) $mg\left(\frac{\rho_0}{\rho}-1\right)$ (d) $mg\left(1+\frac{\rho}{\rho_0}\right)$

9. A total of 48 J heat is given to one mole of helium kept in a cylinder. The temperature of helium increased by 2°C. The work done by the gas is: (Given R = 8.3 JK^{-1} $mole^{-1}$)

   (a) 72.9 J (b) 24.9 J (c) 48 J (d) 23.1 J

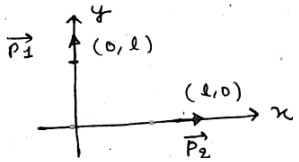
10. Two SHM are represented by the equations $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$, $y_2 = 5\ (\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ ratio of amplitude of y_1 and $y_2 = x : 1$, the value of 'x' is

(a) 2 (b) 1 (c) 4 (d) 6

11. A dipole of moment $\vec{P}$ is placed in a uniform electric field $\vec{E}$ parallel to $\vec{P}$. In surrounding of dipole there exist a spherical equipotential surface, find it's radius:

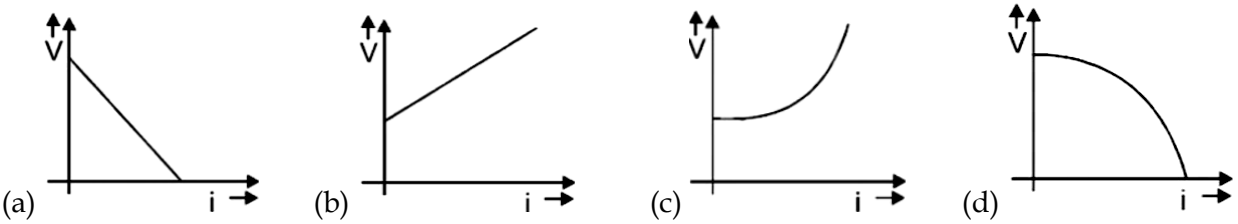
(a) $\left(\frac{KP}{E}\right)^{1/3}$ (b) $\left(\frac{2KP}{E}\right)^{1/3}$ (c) $\left(\frac{KP}{2E}\right)^{1/3}$ (d) $\left(\frac{KP}{5E}\right)^{1/3}$

12. Figure shoes two dipoles P_1 and P_2 in a co-ordinate system. The interaction energy of this system of two dipole:

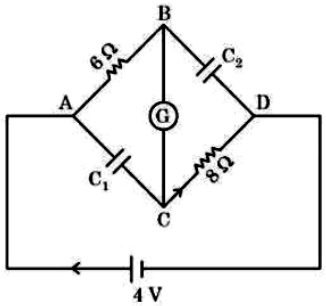


(a) $\frac{2KP_1P_2}{\sqrt{3}\ell^3}$ (b) $\frac{3KP_1P_2}{2\sqrt{2}\ell^3}$ (c) $\frac{3KP_1P_2}{4\sqrt{2}\ell^3}$ (d) $\frac{2KP_1P_2}{3\sqrt{2}\ell^3}$

13. If internal resistance of a cell is proportional to current drawn from the cell. Then the best representation of terminal potential difference of a cell with current drawn from cell will be



14. In this figure the resistance of the coil of galvanometer G is 2Ω. The emf of the cell is 4V. The ratio of potential difference across C_1 and C_2 will be:

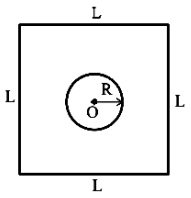


(a) 5/4 (b) ¾ (c) 1 (d) 4/5

15. Two particles X and Y having equal charges are being accelerated through the same potential difference. Thereafter they enter normally in a region of uniform magnetic field and describes circular paths of radius R_1 and R_2 respectively. The mass ratio of 'X' and 'Y' is

(a) $\left(\frac{R_1}{R_2}\right)$ (b) $\left(\frac{R_2}{R_1}\right)$ (c) $\left(\frac{R_2}{R_1}\right)^2$ (d) $\left(\frac{R_1}{R_2}\right)^2$

16. Find the mutual inductance in the arrangement, when a small circular loop of wire of radius 'R' is placed inside a large square loop of wire of side L(L >> R). The loops are co planar and their centers coincides



(a) $M = \frac{\sqrt{2}\mu_0 R}{L^2}$ (b) $M = \frac{2\sqrt{2}\mu_0 R}{L^2}$ (c) $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$ (d) $M = \frac{\sqrt{2}\mu_0 R^2}{L}$

17. Electromagnetic waves travel in a medium with speed 1.5×10^8 m/s. The relative permeability of the medium is 2.0. The relative permittivity will be:

(a) 4 (b) 1 (c) 2 (d) 5

18. A vessel of depth 'd' is half filled with oil of refractive index n_1 and other half is filled with water of refractive index n_2. The apparent depth of this vessel when viewed from above will be

(a) $\frac{2d(n_1+n_2)}{n_1 n_2}$ (b) $\frac{d(n_1+n_2)}{2n_1 n_2}$ (c) $\frac{dn_1 n_2}{2(n_1+n_2)}$ (d) $\frac{dn_1 n_2}{(n_1+n_2)}$

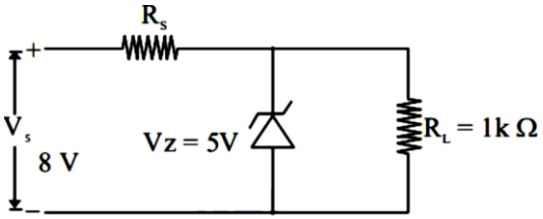
19. In a hypothetical fission reaction

$$92^{x^{236}} \rightarrow 56^{y^{141}} + 36^{z^{92}} + 3R$$

The identity of emitted particles (R) is

(a) Proton (b) Neutron (c) Electron (d) γ-Radiation

20. In the given circuit if the power rating of Zener diode is 10 mW, the value of series resistance R_s to regulate the input unregulated supply is

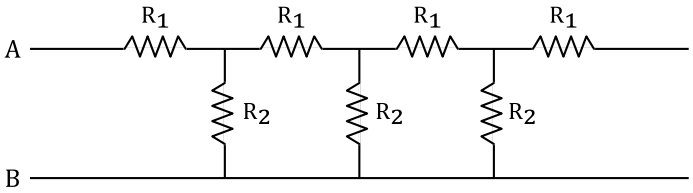


(a) 10 kΩ (b) 10 Ω (c) 1 kΩ (d) $\frac{3}{7}k\Omega < R_s < \frac{3}{5}k\Omega$

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. Consider an infinite ladder shown in figure. A voltage 'V' is applied between the point A and B. This applied value of voltage is halved after each section then value $\left(\frac{R_2}{R_1}\right)$ is



22. A particle initially at rest starts moving from reference point x = 0 along x-axis, with velocity 'v' that varies as $v = 4\sqrt{x}$ m/s. The acceleration of particle in m/s² is

23. A small bob tied at one end of a thin string of length 1m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is _____ m/s. (Take g = 10 m/s²)

24. The first overtone frequency of an open organ pipe is equal to the fundamental frequency of a closed organ pipe. If the length of the closed organ pipe is 20 cm. The length of open organ pipe in cm is:

25. Four particles, each of mass 'M' and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is $\frac{1}{n}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ then value of 'n' is

Chemistry

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is:

   (a) 1186 (b) 84.3 (c) 118.6 (d) 11.86

2. The radius of the second Bohr orbit for hydrogen atom is:

   (Plank's const. h = 6.6262×10^{-34} Js; mass of electron = 9.1091×10^{-31} kg; charge of electron e = 1.60210×10^{-19} C ; permittivity of vaccum $\in_0$ = 8.854185×10^{-12} kg^{-1} m^{-3} A^2)

   (a) 1.65 Å (b) 4.76 Å (c) 0.529 Å (d) 2.12 Å

3. In general, the properties that decrease and increase down a group in the periodic table, respectively, are:

   (a) atomic radius and electronegativity (b) electron gain enthalpy and electronegativity

   (c) electronegativity and atomic radius

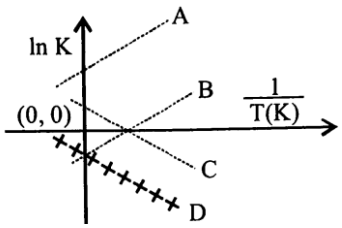
   (d) electronegativity and electron gain enthalpy

4. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^- ?

   (a) Li_2^+ is unstable and Li_2^- is stable (b) Li_2^+ is stable and Li_2^- is unstable

   (c) Both are stable (d) Both are unstable

5. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction?

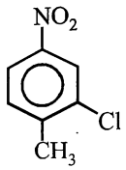


   (a) A and B (b) B and C (c) C and D (d) A and D

6. 20 mL of 0.1 MH_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is: [pK_b of NH_4OH = 4.7]

   (a) 5.2 (b) 9.0 (c) 5.0 (d) 9.4

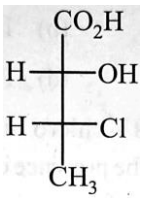
7. The correct IUPAC name of the following compound is:



   (a) 5-chloro-4-methyl-l-nitrobenzene (b) 2-chloro-l-methyl-4-nitrobenzene

   (c) 3-chloro-4-methyl-1-nitrobenzene (d) 2-methyl-5-nitro-1-chlorobenzene

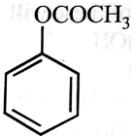
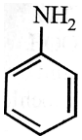
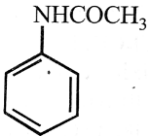
8. The absolute configuration of



is:

(a) (2S, 3S) (b) (2R, 3R) (c) (2R, 3S) (d) (2S, 3R)

9. Which of the following compounds will form significant amount of meta product during mono-nitration reaction?

(a)  (b)  (c)  (d) 

10. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression

(a) $a = \frac{i-1}{(x+y-1)}$ (b) $a = \frac{i-1}{x+y+1}$ (c) $a = \frac{x+y-1}{i-1}$ (d) $a = \frac{x+y+1}{i-1}$

11. The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is : (Molar mass of $PbSO_4$ = 303 g mol^{-1})

(a) 22.8 (b) 15.2 (c) 7.6 (d) 11.4

12. The rate of a reaction doubles when its temperature changes from 300 K to 310 K. Activation energy of such a reaction will be: (R = 8.314 JK^{-1} mol^{-1} and log 2 = 0.301)

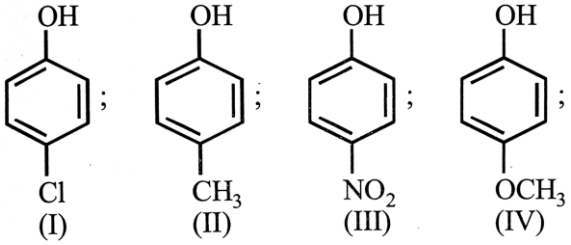
(a) 53.6 kJ mol^{-1} (b) 48.6 kJ mol^{-1} (c) 58.5 kJ mol^{-1} (d) 60.5 kJ mol^{-1}

13. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O)_2(NH_3)]$ respectively are:

(a) +3, +4 and +6 (b) +3, +2 and + 4 (c) +3, 0 and +6 (d) +3, 0 and +4

14. In S_N2 reactions, the correct order of reactivity for the following compounds:
CH_3Cl, CH_3CH_2Cl, $(CH_3)_2CHCl$ and $(CH_3)_3CCl$ is:
(a) $CH_3Cl > (CH_3)_2CHCl > CH_3CH_2Cl > (CH_3)_3CCl$
(b) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$
(c) $CH_3CH_2Cl > CH_3Cl > (CH_3)_2CHCl > (CH_3)_3CCl$
(d) $(CH_3)_2CHCl > CH_3CH_2Cl > CH_3Cl > (CH_3)_3CCl$

15. Arrange the following compounds in order of decreasing acidity:



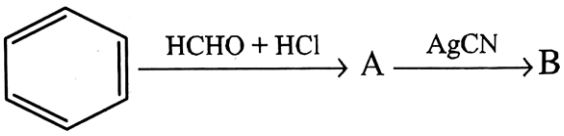
(a) II > IV > I > III (b) I > II > III > IV (c) III > I > II > IV (d) IV > III > I > II

16. The major product of following reaction is:

$$R-C\equiv N \xrightarrow[(ii)\ H_2O]{(i)\ AlH(i\text{-}Bu)_2}$$

(a) RCOOH (b) $RCONH_2$ (c) RCHO (d) RCH_2NH_2

17. The compounds A and B in the following reaction are, respectively:



(a) A = Benzyl alcohol, B = Benzyl cyanide (b) A = Benzyl chloride, B = Benzyl cyanide

(c) A = Benzyl alcohol, B = Benzyl isocyanide (d) A = Benzyl chloride, B = Benzyl isocyanide

18. Which one of the following statements is correct?

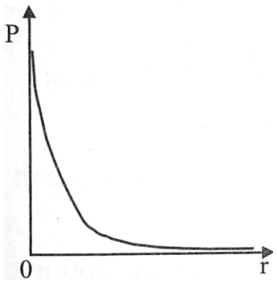
(a) All amino acids except lysine are optically active

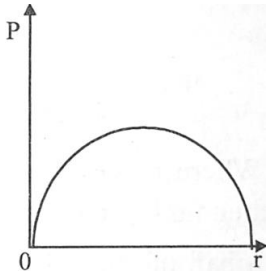
(b) All amino acids are optically active

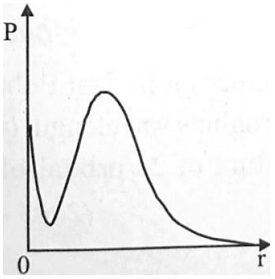
(c) All amino acids except glycine are optically active

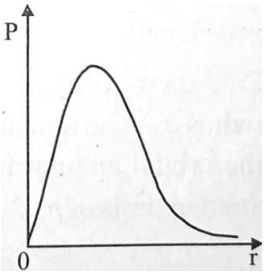
(d) All amino acids except glutamic acids are optically active

19. P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r from the nucleus. The volume of this shell is $4\pi r^2 dr$. The qualitative sketch of the dependence of P on r is

(a) 

(b) 

(c) 

(d) 

20. The molar solubility (in mol L^{-1}) of a sparingly soluble salt MX_4 is 's'. The corresponding solubility product is K_{sp} 's' is given in term of K_{sp} by the relation:

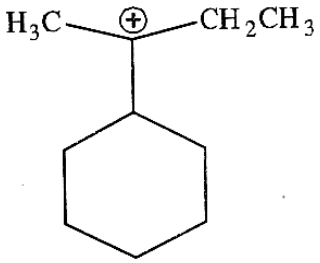
(a) $s = (256\ K_{sp})^{1/5}$ (b) $s = (128\ K_{sp})^{1/4}$ (c) $s = (K_{sp}/128)^{1/4}$ (d) $s = (K_{sp}/256)^{1/5}$

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. Not considering the electronic spin, the degeneracy of the second excited state (n = 3) of H atom is 9, while the degeneracy of the second excited state of H^- is

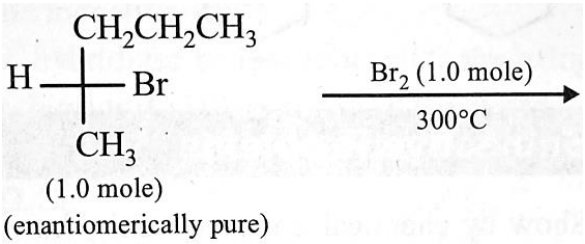
22. The total number of contributing structures showing hyperconjugation (involving C–H bonds) for the following carbocation is



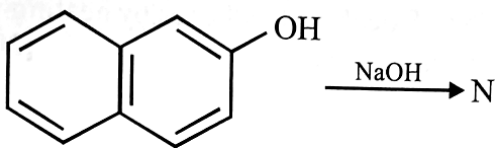
23. A list of species having the formula XZ_4 is given below. XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$ and $[PtCl_4]^{2-}$.

Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is

24. In the following monobromination reaction, the number of possible chiral products is



25. The number of resonance structures for N is



Mathematics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. Let a^2, b^2 and c^2 be three district number in A.P. if ab + bc + ac = 1 then (b + c), (c + a), (a + b) are in

   (a) AP (b) GP (c) HP (d) None of these

2. If $x \in \{1, 2, 3 \ldots\ldots\ldots, 9\}$ and $f_n(x) = x\ x\ x \ldots\ldots\ldots x$ (n-digits) then $(f_n(3))^2 + f_n(2)$ is equal to

   (a) $2f_{2n}(1)$ (b) $f_n^2(1)$ (c) $f_{2n}(1)$ (d) $f_{2n}(4)$

3. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^TQ^{2019}P$ is equal to

   (a) $\begin{bmatrix} 1 & 2019 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{\sqrt{3}}{2} & 2019 \\ 0 & \frac{\sqrt{3}}{2} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{2019}{2} \\ \frac{-2019}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. Let $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function $\forall\ x \in R$, then which of the following must be CORRECT?

   (a) $a^2 - 3b - 15 > 0$ (b) $a^2 - 3b + 15 < 0$ (c) $a^2 - 3b + 25 < 0$ (d) $a^2 - 3b + 15 > 0$

5. The co-ordinate of the point on $y^2 = 8x$ which is closest from $x^2 + (y + 6)^2 = 1$ are

   (a) (2, –4) (b) (18, –12) (c) (32, 16) (d) (32, –16)

6. Solution of $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx} = \left(\frac{x+y+1}{x+y+2}\right)$, if at x = 1, y = 1 is

   (a) $l\,n\left|\frac{(x-y)^2-2}{2}\right| = 2(x+y)$ (b) $l\,n\left|\frac{(x-y)^2-2}{2}\right| = 2(x-y)$

   (c) $l\,n\left|\frac{(x-y)^2+2}{2}\right| = 2(x+y)$ (d) None of these

7. If the line $x\cos\alpha + y\sin\alpha = P$ cuts the circle $x^2 + y^2 = a^2$ at A and B ($0 < P < a$) then the equation of circle, whose one diameter is line segment AB is

   (a) $x^2 + y^2 - a^2 + 2P(x\cos\alpha + y\sin\alpha - P) = 0$ (b) $x^2 + y^2 - a^2 - 2P(x\cos\alpha + y\sin\alpha - P) = 0$

   (c) $x^2 + y^2 - a^2 - 4P(x\cos\alpha + y\sin\alpha + P) = 0$ (d) None of these

8. If the line $\vec{r} = \vec{a} + t\vec{b}$ and $\vec{r} = \vec{c} + \lambda\vec{d}$ are co-planar then

   (a) $(\vec{a} - \vec{b}).(\vec{c} \times \vec{d}) = 0$ (b) $(\vec{c} - \vec{d}).(\vec{a} \times \vec{b}) = 0$

   (c) $(\vec{b} - \vec{d}).(\vec{a} \times \vec{c}) = 0$ (d) $(\vec{a} - \vec{c}).(\vec{b} \times \vec{d}) = 0$

9. Sum of maximum and minimum value of $y = (\sin^{-1}x)^4 + (\cos^{-1}x)^4$ is

(a) $\frac{137\pi^4}{128}$ (b) $\frac{69\pi^4}{64}$ (c) $\frac{37\pi^4}{32}$ (d) $\frac{141\pi^4}{128}$

10. If $f(x) = \int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}}\, dx$ and $f\left(\frac{-\pi}{2}\right) = 0$, then $f\left(\frac{-\pi}{3}\right)$ is equal to

(a) $\frac{-\pi}{3}$ (b) $\frac{-\pi}{6}$ (c) $\frac{-2}{3}\sin^{-1}\left(\frac{1}{\sqrt{8}}\right)$ (d) $\frac{2}{3}\sin^{-1}\left(\frac{1}{\sqrt{8}}\right)$

11. A teacher conducts quiz among the five students of his batch and distributes the answer sheet among them randomly for evaluation then the probability that there are at least two students who are not evaluating their own answer sheet is equal to

(a) $\frac{1}{120}$ (b) $\frac{7}{120}$ (c) $\frac{119}{120}$ (d) $\frac{113}{120}$

12. Let $f : R \to R$ be $f(x) = x^3 + 3$ and $g : R \to R$ be $g(x) = 2x + 1$, then $f^{-1} \circ g^{-1}(23)$ is equal to

(a) 1 (b) 2 (c) $(14)^{\frac{1}{3}}$ (d) $(15)^{\frac{1}{3}}$

13. The shortest distance between z-axis and the line $\frac{x-2}{3} = \frac{y-5}{2} = \frac{z+1}{-5}$ is equal to

(a) $\frac{11}{\sqrt{13}}$ (b) $\frac{17}{\sqrt{13}}$ (c) $\frac{11}{13}$ (d) $\frac{\sqrt{11}}{13}$

14. $\lim\limits_{x\to\infty} \frac{\sum\limits_{r=1}^{10}(x+r)^{2010}}{(x^{1006}+1)(2x^{1004}+1)}$ is equal to

(a) $\frac{1}{2}$ (b) 1 (c) 5 (d) 1005

15. Let matrix $A = \begin{bmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$, where $x, y, z \in N$.

If $|(adj(adj(adj(adjA))))| = 4^8.5^{16}$, then number of such matrix A is equal to

(a) 28 (b) 36 (c) 55 (d) 66

16. If $\int_0^a f(2a-x)dx = 4$ and $\int_0^a f(x)\,dx = 2$, then $\int_0^{2a} f(x)\,dx$ is equal to

(a) 2 (b) 4 (c) 6 (d) 8

17. If $|z - 1 - i| = 1$ then the locus of a point represented by the complex number $5(z - i) - 6$ is

(a) A circle with centre (1, 0) and radius 3 (b) A circle with centre (–1, 0) and radius 5

(c) Line passes through origin (d) Line passes through (–1, 0)

18. Let A and B are two event of a random experiment such that P(A) = $\frac{1}{4}$, $P\left(\frac{B}{A}\right)=\frac{1}{2}$ and $P\left(\frac{A}{B}\right)=\frac{1}{4}$, then $P\left(\frac{\bar{A}}{\bar{B}}\right)$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

19. Distance of origin from plane containing lines

$L_1 : \bar{r}=(2+\lambda)\hat{i}+(1+7\lambda)\hat{j}+(-2-5\lambda)\hat{k}$ and $L_2 : \bar{r}=(4+\mu)\hat{i}+(-3+\mu)\hat{j}-\mu\hat{k}$ is

(a) $\frac{1}{\sqrt{14}}$ (b) $\frac{2}{\sqrt{14}}$ (c) $\frac{3}{\sqrt{14}}$ (d) $\frac{4}{\sqrt{14}}$

20. $\int_0^{\frac{\pi}{2}} \frac{x\sin 2x}{\sin^4 x+\cos^4 x}dx$ is equal to

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{8}$ (c) $\frac{\pi^2}{16}$ (d) None of these

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. The value of $\lim_{x\to\infty}\left(\frac{1}{n^n}\prod_{r=1}^{n}(n+r)\right)^{\frac{1}{n}}$ is $\frac{\lambda}{e}$. Find value of λ.

22. Let $f(x) = \log_e x + 2x^3 + 3x^5$, where $x > 0$ and $g(x)$ is the inverse function of $f(x)$, then $g'(5)$ is equal to $\frac{a}{b}$ find value of (a + b)

23. Let S is the locus of variable point z in complex plane, such that $\left|\frac{2z+i}{z+2i}\right|\le 1$, then area of region bounded by $\frac{\pi}{a}$. Find value of a.

24. Length of the latus rectum of the conic described parametrically as $y = 10t + 4$, $x = 5t^2 + 2$ is

25. Number of terms free from radical sign in the expansion of $(\sqrt{5}+\sqrt[4]{11})^{100}$ is

Answer – key

Physics

1.	b
2.	c
3.	c
4.	b
5.	c
6.	a
7.	b
8.	a
9.	d
10.	b
11.	a
12.	c
13.	d
14.	d
15.	d
16.	c
17.	c
18.	b
19.	b
20.	d
21.	2
22.	8
23.	5
24.	80
25.	2

Chemistry

1.	b
2.	d
3.	c
4.	c
5.	a
6.	b
7.	b
8.	d
9.	c
10.	a
11.	c
12.	a
13.	c
14.	b
15.	c
16.	c
17.	d
18.	c
19.	d
20.	d
21.	3
22.	6
23.	4
24.	5
25.	9

Math

1.	c
2.	c
3.	a
4.	c
5.	a
6.	d
7.	b
8.	d
9.	a
10.	c
11.	c
12.	b
13.	a
14.	c
15.	b
16.	c
17.	b
18.	a
19.	b
20.	b
21.	4
22.	23
23.	1
24.	20
25.	26

Solutions

Physics

(1) We may round off the number a to 1 (for $a \le 5$) and to 10 (for $5 < a \le 10$)

$a \times 10^b \longrightarrow$ 'b' is order if $a \le 5$

if $a > 5$ then order will be $(b+1)$.

(2) $T \propto G^a m^b r^c$

$[T] = [M^{-1}L^3T^{-2}]^a [M]^b [L]^c$

$[T] = [M]^{-a+b} [L]^{3a+c} [T]^{-2a}$

$-2a = 1 \Rightarrow a = -1/2$

$-a + b = 0 \Rightarrow b = -1/2$

$3a + c = 0$

$c = -3a$

$c = -3\{-1/2\}$

$c = 3/2$

$T = K G^{-1/2} m^{-1/2} r^{3/2}$

$T^2 = K' G^{-1} m^{-1} r^3$

$T^2 \propto \frac{r^3}{GM}$ → Option (c)

(3) $|\vec{x}| = |\vec{y}| = r_0$

$|\vec{x} - \vec{y}| = n|\vec{x} + \vec{y}|$

$\sqrt{x^2 + y^2 - 2xy\cos\theta} = n\sqrt{x^2 + y^2 + 2xy\cos\theta}$

$x^2 + y^2 - 2xy\cos\theta = n^2(x^2 + y^2 + 2xy\cos\theta)$

$2r_0^2 - 2r_0^2\cos\theta = n^2(2r_0^2 + 2r_0^2\cos\theta)$

$2r_0^2 - 2r_0^2 n^2 = 2r_0^2 n^2\cos\theta + 2r_0^2\cos\theta$

$1 - n^2 = \cos\theta(n^2 + 1)$

$\cos\theta = \left(\frac{1-n^2}{n^2+1}\right) \Rightarrow \cos\theta = \left(\frac{n^2-1}{-n^2-1}\right)$

$\theta = \cos^{-1}\left\{\frac{n^2-1}{-n^2-1}\right\}$

(4) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ $\quad \vec{B} = \hat{i} + \hat{j}$

Projection of $\vec{A}$ along $\vec{B} = \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|}\hat{B} = \left(\frac{2}{\sqrt{2}}\right)\hat{B}$

$= \frac{2}{\sqrt{2}}\left\{\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right\} = (\hat{i} + \hat{j})$

(5)

$H_1 = \frac{(\sqrt{3}u)^2\sin^2\theta}{2g}$ $\quad H_2 = \frac{(\sqrt{2}u)^2\sin^2 60°}{2g}$

$\frac{3u^2\sin^2\theta}{2g} = \frac{2u^2(\sqrt{3}/2)^2}{2g} \Rightarrow \sin^2\theta = 2\left\{\frac{3}{4}\right\}$

$\sin^2\theta = 1/2 \Rightarrow \theta = 45°$

(6) $P = \text{const}$

$FV = \text{const}$

$mav = \text{const}$

$aV = \text{const}$

$v\frac{dv}{dt} = c$

$\int v\,dv = \int c\,dt$

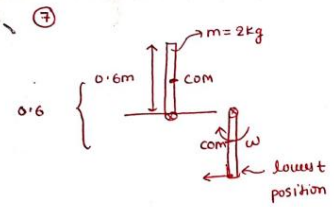
$\frac{v^2}{2} = ct$

$v \propto t^{1/2}$

$\frac{ds}{dt} \propto t^{1/2}$

$s \propto t^{3/2}$

(7)



$mgh = \frac{1}{2}I\omega^2$

$2(10)(0.6) = \frac{1}{2}\left(\frac{2(0.6)^2}{3}\right)\omega^2$

$20 = \frac{(0.6)}{3} = \omega^2$

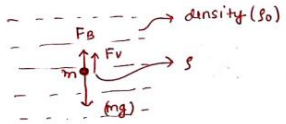
$\omega^2 = \frac{60}{0.6} = 100$

$\omega = 10$

$v = R\omega$

$v = (0.6)(10) = 6\,m/s$

(8)



$F_B + Fv = mg$

$(V\rho_0 g) + Fv = mg$

$\frac{m}{\rho}\rho_0 g + Fv = mg$

$Fv = mg - \frac{\rho_0}{\rho}mg$

$Fv = mg\left(1 - \frac{\rho_0}{\rho}\right)$

$\rho = \frac{m}{V} \Rightarrow V = \left(\frac{m}{\rho}\right)$

(9) $Q = \Delta U + W$

$Q = 48\,J$

$\Delta U = \frac{f}{2}nR\Delta T = \frac{5}{2}(1)\left(\frac{25}{5}\right)X2$

$\Delta U = 25\,J$

$Q = \Delta U + W$

$W = Q - \Delta U = 48 - 25 = 23\,J$

$W = 23\,J$

(10) $y_1 = 10\sin(3\pi t + \pi/3)$

$y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

$y_2 = 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right) = 10\{\sin 3\pi t \cdot \cos\pi/3 + \cos 3\pi t \cdot \sin\pi/3\}$

$y_2 = 10\{\sin(3\pi t + \pi/3)\}$

Amplitude of both wave = $(y_1)A = 10$, $(y_2)A = 10$

Ratio = 1 : 1

$x : 1$

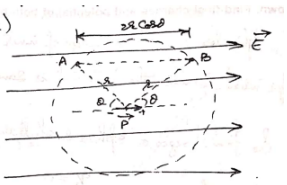
$x = 1$

11.

Sol: Electric Pot'l at pts A & B are – (due to dipole)

$V_A = -\frac{kp\cos\theta}{r^2}$ & $V_B = \frac{kp\cos\theta}{r^2}$

Due to dipole $(V_B - V_A)_D = \frac{2kp\cos\theta}{r^2}$



Pot'l diff between pts A and B due to EF is

$(V_A - V_B)_{ext} = Ed = E(2r\cos\theta)$

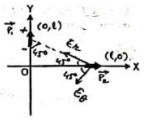
for A and B to be at same pot'l we use $(V_B - V_A)_D = (V_A - V_B)_{ext}$

$\frac{2kp\cos\theta}{r^2} = E(2r\cos\theta)$

$r = \left(\frac{kp}{E}\right)^{1/3}$ Ans.

12.

Sol: IE of a dipole in an EF is given as $U=-\vec{P}.\vec{E}$



Due to $\vec{P}_1$, EF Components at (1,0) are -

$E_2 = \frac{2kP_1\cos 45^\circ}{(\sqrt{2}L)^3}$ and $E_\theta = \frac{kP_1\sin 45^\circ}{(\sqrt{2}L)^3}$

$E_2 = \frac{kP_1}{2L^3}$ $\quad E_\theta = \frac{kP_1}{4L^3}$

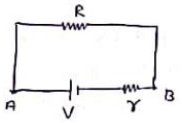
Interaction energy P_2 in the EF of $\vec{P}_1$ is

$U = -\vec{P}_2.\vec{E}_1 = -\vec{P}_2.(\vec{E}_2+\vec{E}_\theta)$

$= -P_2\left(\frac{kP_1}{2L^3}\right)\left(-\frac{1}{\sqrt{2}}\right) - P_2\left(\frac{kP_1}{4L^3}\right)\left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{3}{4}\frac{kP_1P_2}{\sqrt{2}L^3}$ Ans.

(13)



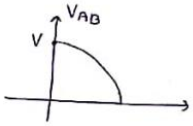
$V_A - V_B = V - ir$

$V_{AB} = V - i(ki)$

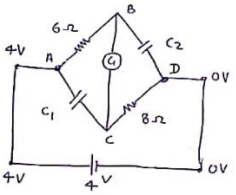
$V_{AB} = V - ki^2$

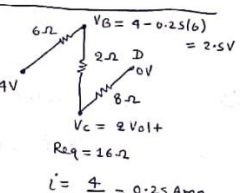
Potential diff across terminal — emf — current

$V_{AB} = -ki^2 + V$



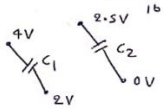
(14)





$R_{eq} = 16\Omega$

$i = \frac{4}{16} = 0.25$ Amp



$\Delta V_{C_1} = 2V$

$\Delta V_{C_2} = 2.5V$

$\frac{\Delta V_{C_1}}{\Delta V_{C_2}} = \frac{2}{2.5} = \frac{4}{5}$ Volt

(15)

for X and Y

↓ q ↓ q

Energy after accln

$X = (qV)$ $\quad Y = qV$

Radius $= R = \frac{mv}{qB}$

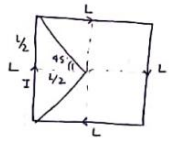
$R = \frac{\sqrt{2kEm}}{qB}$

$R \propto \sqrt{m}$

$\frac{R_1}{R_2} = \sqrt{\frac{m_X}{m_Y}}$

$\frac{m_X}{m_Y} = \frac{R_1^2}{R_2^2}$

(16)



Magnetic flux through circular loop due to current in wire $= \phi = Mi$

Fluid due to square loop at center

$= 4\left\{\frac{\mu_0 I}{4\pi L}\{\sin 45^\circ + \sin 45^\circ\}\right\}$

$B = \frac{\mu_0 I}{\pi L}\sqrt{2}$

$\phi = \frac{\mu_0 I}{\pi L} \times \pi R^2 = \frac{\mu_0 I \sqrt{2} R^2}{L} = Mi$

$M = \frac{\sqrt{2}\mu_0 R^2}{L}$

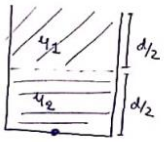
(17)

$V = 1.5\times10^8$ m/s

$V = \frac{c}{\mu} \Rightarrow \mu = \frac{3\times10^8}{1.5\times10^8} = 2$

$\sqrt{\mu_r \varepsilon_r} = 2 \Rightarrow \sqrt{2\varepsilon_r} = 2 \Rightarrow \boxed{\varepsilon_r = 2}$

(18)



apparent depth $= \frac{d/2}{\mu_1} + \frac{d/2}{\mu_2}$

$= \frac{d}{2}\left\{\frac{1}{\mu_1} + \frac{1}{\mu_2}\right\}$

$= \frac{d}{2}\left\{\frac{\mu_2+\mu_1}{\mu_1\mu_2}\right\}$

$\boxed{\text{app depth} = \frac{d(\mu_1+\mu_2)}{2\mu_1\mu_2}}$

(19)

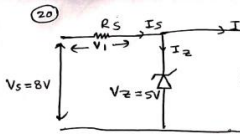
$_{92}X^{236} \longrightarrow {}_{56}Y^{141} + 36Z^{92} + 3R$

No of proton is Not changing

Mass no is decreased by (3)

So R → Neutron.

(20)



Pd across $R_S = V_1 = 3V$

current through load

$I = \frac{5}{1000} = 5mA$

max^m current through zener diode

$I_{Z\,max} = \frac{10}{5} = 2mA$

min^m current through diode $= 0$

$I_{S\,max} = 5+2 = 7mA$

$R_{S\,min} = \frac{V_1}{I_{S\,max}} = \frac{3}{7}$ kΩ

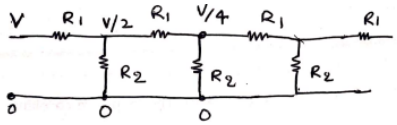
Similarly

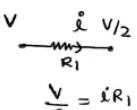
$I_{S\,min} = 5mA$

$R_{S\,max} = \frac{V_1}{I_{S\,min}} = \frac{3}{5}$ kΩ

$\frac{3}{7}$ kΩ $< R_S \le \frac{3}{5}$ kΩ

(21)





$\frac{V}{2} = iR_1$ —(i)

$i = \left(\frac{V}{2R_1}\right)$

$i_1 = \left(\frac{V}{2R_2}\right)$

$i_2 = \left(\frac{V}{4R_1}\right)$

$i = i_1 + i_2$

$\frac{V}{2R_1} = \frac{V}{2R_2} + \frac{V}{4R_1} \Rightarrow \frac{V}{4R_1} = \frac{V}{2R_2}$

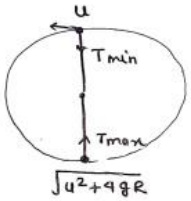
$R_2 = 2R_1 \Rightarrow \boxed{\frac{R_2}{R_1} = 2}$

(22) $V = 4\sqrt{x}$

$a = v\frac{dv}{dx} = 4\sqrt{x}\left\{4\frac{1}{2\sqrt{x}}\right\}$

$\boxed{a = 8\ m/s^2}$

(23)



$T_{max} = mg + \frac{mv^2}{R}$

$T_{max} = mg + \frac{m(u^2+4gR)}{R}$

$T_{min} = -mg + \frac{mu^2}{R}$

$\frac{T_{max}}{T_{min}} = \frac{5}{1} = \frac{g + \{u^2+4gR/R\}}{g + u^2/R}$

$\boxed{u = 5\ m/s}$

(24)

$f_1 = \frac{2V}{2l_1}$

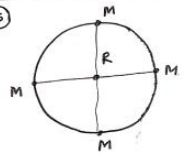
$f_2 = \frac{V}{4l_2}$

$f_1 = f_2$ (given) $\Rightarrow l_1 = 4l_2$

$\boxed{l_1 = 80cm}$

$\frac{2V}{2l_1} = \frac{V}{4l_2}$

(25)



$F_{net} = \frac{GMm}{(2R)^2} + \frac{GMm}{(R\sqrt{2})^2}\cos 45° + \frac{GMm}{(R\sqrt{2})^2}\cos 45°$

$\frac{mu^2}{R} = \frac{GMm}{R^2}\left\{\frac{1+2\sqrt{2}}{4}\right\}$

$\boxed{u = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}}$

Chemistry

Q.1) (b)

Ans) Given - $M_2CO_3 + 2HCl \rightarrow 2MCl + H_2O + CO_2$

1 gm — 0.01186 mole

From above equation

$n_{M_2CO_3} = n_{CO_2}$

$\therefore \frac{1}{\text{Molar mass of } M_2CO_3} = 0.01186$

$\therefore$ Molar mass of $M_2CO_3 = \frac{1}{0.01186}$

$\therefore \boxed{M = 84.3\ gm/mole}$

Q.2) (d)

Ans - Radius of n^{th} Bohr orbit in H-atom $= 0.53\ n^2\ A°$

Radius of 2nd Bohr orbit $= 0.53 \times (2)^2$

$\boxed{= 2.12\ A°}$

Q.3) (c)

Ans) Electronegativity $\propto \frac{1}{\text{Size}}$

Q.4) (c)

Ans) Bond order of $Li_2^+ = \frac{1}{2}(3-2) = \frac{1}{2}$

Bond order of $Li_2^- = \frac{1}{2}(4-3) = \frac{1}{2}$

Q.5) (a)

As $\ln K = \frac{-\Delta H}{RT} + \frac{\Delta S°}{R}$

for exothermic reaction.

ΔH = (-)ve, slope $= \frac{-\Delta H°}{R}$ = (+)ve

$\therefore$ from graph line should be A & B

Q.6) (b)

Ans) m. mole of $H_2SO_4 = 20 \times 0.1 = 2$

m. mole of ~~NaOH~~ $NH_4OH = 30 \times 0.2 = 6$

$H_2SO_4 + 2NH_4OH \rightarrow (NH_4)_2SO_4 + H_2O$

initial	2 m mole	6 m mole	0
final	(2-2)	(6-2×2)	2 m mole
	= 0 m mole	= 2 m mole	

$[NH_4OH]_{left} = 2$ m mole

$[(NH_4)_2SO_4] = 2$ m mole

$[NH_4^+] = 2 \times 2 = 4$ m mole

Total volume $= 30 + 20 = 50$ ml

$pOH = pK_b + \log\left[\frac{Salt}{Base}\right]$

$= 4.7 + \log\frac{4/50}{2/50}$

$= 4.7 + \log 2 = 5$

$pH = 14 - pOH$

$pH = 14 - 5 = 9$

$\boxed{pH = 9}$

Q.7) (b)

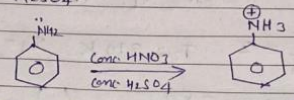
Ans) 2- chloro - 1 - methyl - 4- nitrobenzene

Q.8) (d)

Ans)

Q.9) (c)

Ans) Nitration takes place in presence of conc. HNO_3 + conc. H_2SO_4.



$-NH_2$ gp. o, p- directing

$-NH_3^{\oplus}$ gp. m-directing

Q.10) (a)

Ans) $A_xB_y \rightleftharpoons xA^{y+} + yB^{x-}$

t=0	1	0	0
teq.	1-α	xα	yα

Total No. of mole (i) $= 1 - \alpha + x\alpha + y\alpha$

$i - 1 = x\alpha + y\alpha - \alpha \Rightarrow \alpha(x+y-1)$

$\therefore \boxed{\alpha = \frac{i-1}{(x+y-1)}}$

Q.11) (c)

Ans) Half cell reaction $PbSO_4 \rightarrow Pb^{4+} + 2e^-$

According to reaction : $PbSO_4 \rightarrow Pb^{4+} + 2e^-$

∵ we require 2F for electrolysis of 1 mole or 303 gm of $PbSO_4$

∴ Amount of $PbSO_4$ electrolysed by 0.05F

$= \frac{303}{2} \times 0.05 = 7.57\ gm \simeq \boxed{7.6\ gm}$

Q.12) (a)

Ans)
$$\frac{\log K_2}{\log K_1} = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

given $\frac{K_2}{K_1} = 2$, $T_2 = 310\ K$, $T_1 = 300\ K$

$$\therefore \log 2 = \frac{-E_a}{2.303 \times 8.314}\left(\frac{1}{310} - \frac{1}{300}\right)$$

$E_a = 53598.6\ J/mole$

$\boxed{E_a = 53.6\ Kj/mole}$

Q.13) (c)

Ans) +3, 0, +6

Q.14) (b)

Ans) Steric congestion around the carbon atom undergoing the inversion process will slow down S_N2 reaction

$CH_3Cl > CH_3CH_2-Cl > (CH_3)_2CH_2-Cl > (CH_3)_3CCl$

Q.15) (c)

Ans) Electrong withdrawing gp increases the acidity and electron releasing gp decreases Acidic nat

Q.16) (c)

Ans) $R-C\equiv N \xrightarrow[(ii)\ H_2O]{(i)\ AlH(i\text{-}Bu)_3} R-CHO$

Q.17) (d)

Ans) A = Benzyl chloride , B = Benzyl isocyanide.

Q.18) (c)

Ans)

Q.19) (d)

Ans) Radial probability function curve for 1s is (d)

Here P_r is $4\pi r^2 R^2$

Q.20) (d)

Ans) $MX_4 \rightleftharpoons M^{4+} + 4X^-$

$S \quad\quad 4S$

$K_{sp} = [S][4S]^4 \Rightarrow 256\,S^5$

$S = \left(\frac{K_{sp}}{256}\right)^{1/5}$

Q.21) (3)

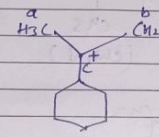
Ans) Ground state configuration

H^- : 1s 2s 2p

In second excited state, electron will jump from 1s to 2p so degeneracy of second excited state of H^- is 3

Q.22) (6)

Ans)



a = 3 hyper conjugative H

b = 2 hyper conjugative H

c = 1 hyper conjugative H

Q.23) (4)

Ans)

XeF_4 → Square planar (sp^3d^2)

SF_4 → See-saw (sp^3d)

SiF_4 → Tetrahedral (sp^3)

BF_4^- → Square planar (sp^3d^2)

$[Cu(NH_3)_4]^{2+}$ → Square planar (dsp^2)

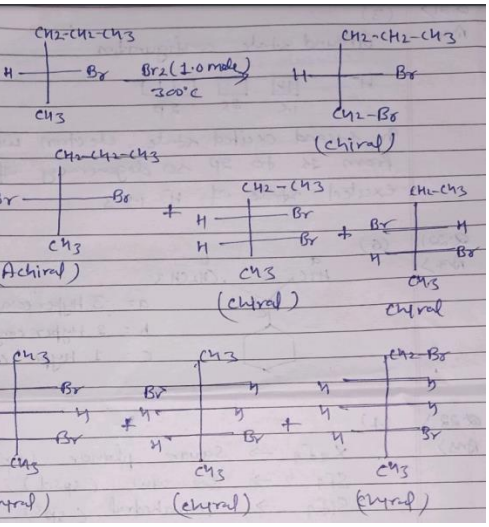
$[FeCl_4]^{2-}$ → Tetrahedral (sp^3)

$[CoCl_4]^{2-}$ → Tetrahedral (sp^3)

$[PtCl_4]^{2-}$ → Square planar (dsp^2)

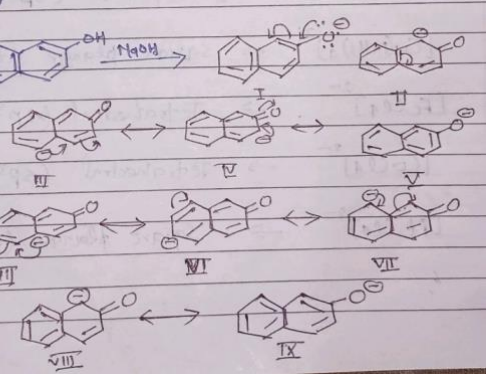
Q.24) (5)

Ans)



Q.25) (9)

Ans)



Math

1. [C] a^2, b^2, c^2 — AP

↳ $a^2+ab+bc+ca,\ b^2+ab+bc+ca,\ c^2+ab+bc+ca$ — AP

↳ $(a+b)(a+c),\ (a+b)(b+c),\ (b+c)(c+a)$ — AP

↳ $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ — AP

↳ $b+c,\ c+a,\ a+b \longrightarrow$ AP

2. [C] $\left(f_n(3)\right)^2 + 2f_n(1) = 9f_n^2(1) + 2f_n(1)$

$= 9\left(\frac{10^n-1}{9}\right)^2 + \frac{2(10^n-1)}{9}$

$= \frac{10^{2n}-1}{10-1} = f_{2n}(1)$

3. [A] $PP^T = \begin{bmatrix} \frac{\sqrt3}{2} & \frac12 \\ -\frac12 & \frac{\sqrt3}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt3}{2} & -\frac12 \\ \frac12 & \frac{\sqrt3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\boxed{PP^T = I}$

$Q^2 = PAP^TPAP^T = PA^2P^T$

↳ $P^TQ^{2019}P = A^{2019} = \begin{bmatrix} 1 & 2019 \\ 0 & 1 \end{bmatrix}$

4. [C] $f'(x) = 3x^2+2ax+b-5\sin 2x > 0 \quad \forall x \in R$

↳ $a^2-8(b-5)<0$

$a^2-3b+15<0$

5. [A] Let the point be $(2t^2, 4t)$

Equation of Normal is $tx+y = 4t+2t^3$

↳ $2t^3+4t+6=0$

↳ $t^3+2t+3=0$

↳ $(t+1)(t^2-t+3)=0$

$\boxed{t=-1}$ Pt be $(2,-4)$

6. [D] Put $x+y=t$

$1+\frac{dy}{dx} = \frac{dt}{dx}$

$\left(\frac{t-1}{t-2}\right)\frac{dy}{dx} = \left(\frac{t+1}{t+2}\right)$

$\left(\frac{t^2+t-2}{t^2+2}\right)dt = 2dx$

$t + \frac{\ln|t^2-2|}{2} = 2x+C$

$(y-x) + \ln\left|\frac{(x+y)^2-2}{2}\right| = C$

7. [B] Let the circle be

$x^2+y^2-a^2+\lambda(x\cos\alpha+y\sin\alpha-p)=0$

$C \equiv \left(-\frac{\lambda\cos\alpha}{2}, -\frac{\lambda\sin\alpha}{2}\right)$

$\lambda = -2p$

8. [D]

9. [A] $f(x) = (\sin^2x)^4 + (\cos^2x)^4$

$f'(x) = 4\frac{(\sin^2x)^3 - (\cos^2x)^3}{\sqrt{1-7x^2}}$

$f(x)$ is decreasing in $(-1, \frac{1}{\sqrt2})$ & increasing in $(\frac{1}{\sqrt2}, 1)$

$f_{max} = f(-1) = \frac{17\pi}{16}^4$, $f_{min} = f(\frac{1}{\sqrt2}) = \frac{\pi^4}{128}$

10. [C] $f(x) = \int\sqrt{\frac{\cos x - \cos^3x}{1-\cos^3x}}\,dx = \int\sqrt{\frac{\cos x}{1-\cos^3x}}\cdot\sin x\,dx$

$f(-\frac{\pi}{2}) = -\frac23\sin^{-1}(\frac12)^{3/2}$

$\cos x = t$, $-\sin x\,dx = dt$

$= -\int dt\,\frac{\sqrt t}{\sqrt{1-(t^{3/2})^2}}$

$f(x) = -\frac23\sin^{-1}(\cos^{3/2}x) + k$

$f(-\frac{\pi}{2}) = -\frac23\sin^{-1}(0) + k$

$k=0$

$= \frac23\cos^{-1}(t^{3/2}) + C$

$= -\frac23\sin^{-1}(t^{3/2}) + k$

11. [C] R.I = 1 − All students are evaluating their own answer sheet

$= 1 - \frac{1}{120} = \frac{119}{120}$

12. [B] $g(f(x)) = g(x^3+8) = 2x^3+7$

$g(f(2)) = 2\cdot2^3+7 = 23$

$f^{-1}(g^{-1}(23)) = 2$

correct option (B) 8 → 2

13. [A] z-Axis is $\vec r = \vec 0 + \lambda(\hat k)$

Line $\equiv \vec r = (2\hat i+5\hat j-\hat k) + \lambda(3\hat i+2\hat j-5\hat k)$

Shortest Distance:

$\left|\frac{(2\hat i+5\hat j-\hat k)\cdot((3\hat i+2\hat j-5\hat k)\times\hat k)}{|(3\hat i+2\hat j-5\hat k)\times\hat k|}\right| = \frac{11}{\sqrt{13}}$

14. [C]

$\lim_{x\to\infty}\frac{(x+1)^{2010}+(x+2)^{2010}+\cdots+(x+10)^{2010}}{(x^{1006}+1)(2x^{1004}+1)} = \frac{10}{2} = 5$

15. [B] $|A|^2{}^4 = (2\cdot5)^{16} \Rightarrow \boxed{|A| = \pm10}$

$|A| = x+y+z$, $x,y,z \in N$

No of Soln $= {}^{9+1}C_{3-1} + {}^{7+3-1}C_{3-1} + {}^{10}C_{3-1}$

$= 36$

$x+y+z=10$, $x\ge1, y\ge1, z\ge1$

$z_1+y_1+z_1=7$, $x_1\ge0, y_1\ge0, z_1\ge0$

16. [C]

$\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_a^{2a} f(x)\,dx$

$x = 2a-t$

$dx = -dt$

$= \int_0^a f(x)\,dx - \int_a^0 f(2a-t)\,dt$

$= \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx = 2+4 = 6$

17. [B] Let $w = 5(z-\bar{i}) = 6$

$\Rightarrow w+1 = 5(z-\bar{i}-1)$

$\Rightarrow |w+1| = 5|z-\bar{i}-1| = 5$

Locus of w is a circle with centre $(-1,0)$ & $r=5$

18. [A] $P(A)\cdot P\left(\frac{B}{A}\right) = P(B)\cdot P\left(\frac{A}{B}\right)$

$\frac{1}{4}\times\frac{1}{2} = P(B)\times\frac{1}{4} \Rightarrow P(B) = \frac{1}{2}$, $P(A) = \frac{1}{4}$ & $P(A\cap B) = \frac{1}{8}$

$P\left(\frac{\bar{A}}{B}\right) = \frac{P(\bar{A}\cap B)}{P(B)} = \frac{1-P(A\cup B)}{1-P(B)} = \frac{1-\left(\frac{1}{2}+\frac{1}{4}-\frac{1}{8}\right)}{1-\frac{1}{2}} = \frac{3}{4}$

19. [B]

20. [B]

$$I = \int_0^{\pi/2} \frac{x\sin 2x}{\sin^4 x+\cos^4 x}\,dx \quad -(1)$$

$$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)\sin 2x}{\cos^4 x+\sin^4 x} \quad -(2)$$

(1)+(2)

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\sin 2x}{1-\frac{1}{2}(1-\cos^2 2x)}\,dx$$

$$= \frac{\pi}{2}\int_0^{\pi/2} \frac{\sin 2x}{\frac{1}{2}(1+\cos^2 2x)}\,dx \qquad I = \pi^2/8$$

21. [4]

$$L = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right)^{1/n}$$

$$\ln L = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\ln\left(1+\frac{r}{n}\right)$$

$$= \int_0^1 \ln(1+x)\,dx \qquad \boxed{L = 4/e}$$

Use By Parts

22. [23]

$$g'(f(x)) = \frac{1}{f'(x)}$$

$\Rightarrow g'(\ln x+2x^3+3x^5) = \frac{1}{\frac{1}{x}+6x^2+15x^4}$

Put $x=1$

$g'(5) = \frac{1}{1+6+15} = \frac{1}{22} = \frac{a}{b}$

$a+b = \boxed{22+1} = 23$

23. [1]

$|(2x)+i(2y+1)|^2 \le |x+i(y+2)|^2$

$\Rightarrow 4x^2+4y^2+4y+1 \le x^2+y^2+4y+4$

$\Rightarrow 3x^2+3y^2 \le 3$

$x^2+y^2 \le 1$

Area $= \pi(1)^2 = \pi$

24. [32]

$$\left(\frac{y}{2}-4\right)^2 = 100\left(\frac{x-2}{5}\right)^2 = 100\left(\frac{x-2}{5}\right)$$

$$\left(\frac{y}{2}-4\right)^2 = 4\cdot 5(x-2)$$

Length of L.R $= 4\cdot 5 = 20$

25 → 26

$$T_{r+1} = {}^{100}C_r\, 5^{\frac{100-r}{2}}\cdot 11^{r/4}$$

r must be: $0, 4, 8, \ldots 100$

$N = 26$